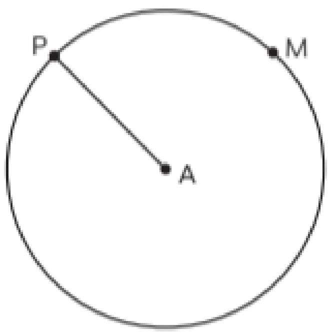
OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. **Given below is a circular park with centre A. Madhav walks at a uniform speed of 0.5 m/s from gate P and reached the centre of the park in 150 seconds.**



What is the straight line distance between the centre of the park and gate M?

(a) 300 m **(b) 150 m**

(c) 2.5 m **(d) 75 m** **[Diksha]**

Ans. *(d) 75 m*

Explanation: As we know,

Distance = Speed × Time

= 0.5 m/s × 150 seconds

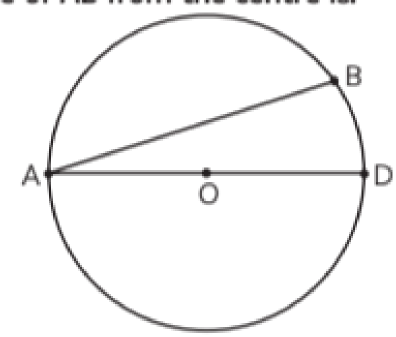
Distance = 75 m

So, AP = 75 m

Hence, the straight line distance between the centre of the park to gate M *i.e.,* AM

AM = AP = 75 m

2. **If AD = 8.2 cm and AB = 8 cm, then the distance of AB from the centre is:**



(a) 9 cm **(b) 0.09 cm**

(c) 0.009 cm **(d) 0.9 cm**

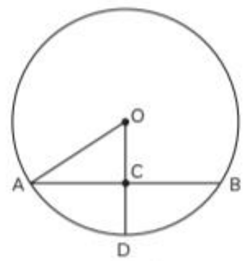
[British Council 2022]

Ans. *(d) 0.9 cm*

Explanation: $d^2 + 4^2 = 4.1^2$

$$d^2 = (4.1)^2 - (4)^2$$
$$d^2 = 16.81 - 16$$
$$d^2 = 0.81$$
$$d = \sqrt{0.81} = 0.9 \text{ cm}$$

3. In the given figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:



(a) 2 cm **(b) 3 cm**
(c) 4 cm **(d) 5 cm**

[NCERT Exemplar]

Ans. *(a) 2 cm*

Explanation: As we know, perpendicular from the centre to a chord, bisects the chord.

$$AC = BC = \frac{1}{2}AB = BC = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$$

Now, in ΔAOC

$$AO^2 = OC^2 + AC^2$$

[By using Pythagoras Theorem]

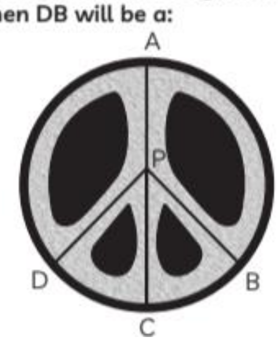
$$OC^2 = AO^2 - AC^2$$
$$OC^2 = 5^2 - 4^2$$
$$= 25 - 16$$
$$OC^2 = 9$$
$$OC = 3 \text{ cm}$$

Now,

$$OA = OD \text{ [Same radius of a circle]}$$
$$OD = 5 \text{ cm}$$

Hence, $CD = OD - OC = (5 - 3) \text{ cm} = 2 \text{ cm}$

4. Given below is the peace symbol which was designed in 1958 by Gerald Holtom, a professional artist and designer. If we join D and B, then DB will be a:

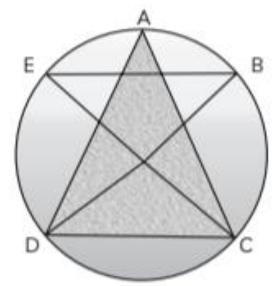


(a) sector **(b) chord**
(c) diameter **(d) radius**

Ans. *(b) chord*

Explanation: DB is a chord. The chord of a circle is a straight line segment whose end points both lie on a circular arc.

5. In the figure shown below, the region DEABC is a:

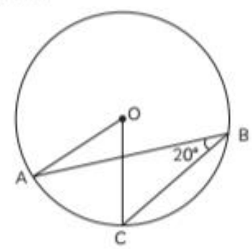


(a) minor arc **(b) major arc**
(c) major segment **(d) minor segment**

Ans. *(c) major segment*

Explanation: The region of the circle between chord and arc is called a segment. Hence, DEABC is a major segment.

6. In the given figure, if $\angle ABC = 20°$, then $\angle AOC$ is equal to:



(a) 20° **(b) 40°**
(c) 60° **(d) 10°**

[NCERT Exemplar]

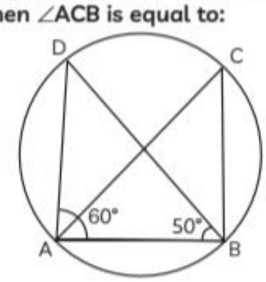
Ans. *(b) 40°*

Explanation: Here, arc AC of the circle making an angle of 20° on the circumference of a circle.

Hence, arc $\angle AOC = 2\angle ABC$

$$\angle AOC = 2 \times 20°$$
$$= 40°$$

7. In the given figure, if $\angle DAB = 60°$, $\angle ABD = 50°$, then $\angle ACB$ is equal to:



(a) 60° (b) 50°
(c) 70° (d) 80°

[NCERT Exemplar]

Ans. *(c) 70°*

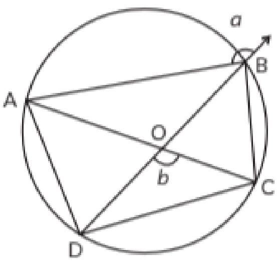
Explanation: In ΔADB,

$$\angle DAB + \angle ABD + \angle ADB = 180^\circ$$
$$60^\circ + 50^\circ + ADB = 180^\circ$$
$$\angle ADB = 180^\circ - 60^\circ - 50^\circ$$
$$\angle ADB = 70^\circ$$

Angles in the same segment of a circle are equal.

Therefore, $\angle ACB = \angle ADB = 70^\circ$

8. Given below is a circle with centre O. Which of the following represents the measure of ∠BCD?



(a) $180^\circ + (a - \frac{b}{2})$ (b) $180^\circ - (a + \frac{b}{2})$
(c) $90^\circ - (a - \frac{b}{2})$ (d) $180^\circ - (a - \frac{b}{2})$

[Diksha]

Ans. *(d)* $180^\circ - (a - \frac{b}{2})$

Explanation: From figure

$\angle AOB = \angle DOC$ [Vertically opposite angles]

Here, $\angle AOB = \angle DOC = b$

Now, $\angle ACB = \frac{1}{2}\angle AOB$

[Angle made by the same segment on the circumference is half of that made on the centre]

So, $\angle ACB = \frac{b}{2}$

Now, DB is a straight line

$\angle ABO + a = 180^\circ$ [Linear pair]

$\angle ABO = 180^\circ - a$

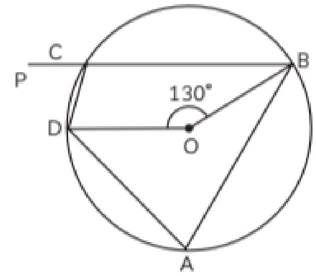
$\angle ACD = \angle ABO = 180^\circ - a$

[Chord AD is Common]

Now, $\angle BCD = \angle ACB + \angle ACD$

$$= \frac{b}{2} + 180^\circ - a$$
$$\angle BCD = 180^\circ - a + \frac{b}{2}$$
$$= 180^\circ - (a - \frac{b}{2})$$

9. Rashi draws a circle and puts 4 points on its circumference. O is the centre of the circle, as shown in the diagram. ∠BAD and ∠BCD are:



(a) $\angle BAD = 65^\circ, \angle BCD = 105^\circ$
(b) $\angle BAD = 50^\circ, \angle BCD = 115^\circ$
(c) $\angle BAD = 50^\circ, \angle BCD = 105^\circ$
(d) $\angle BAD = 65^\circ, \angle BCD = 115^\circ$

Ans. *(d)* $\angle BAD = 65^\circ, \angle BCD = 115^\circ$

Explanation: Since, the arc BCD makes $\angle BOD = 130^\circ$ at the centre and point A is on the circumference forming $\angle BAD$.

$$\Rightarrow \angle BOD = 2\angle BAD$$
$$\Rightarrow \angle BAD = \frac{1}{2}\angle BOD$$
$$\Rightarrow \angle BAD = \frac{1}{2} \times 130^\circ$$
$$\Rightarrow \angle BAD = 65^\circ$$

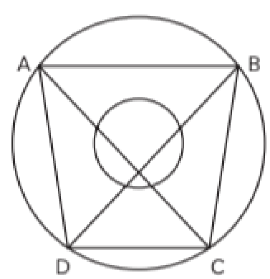
Again, arc BAD makes reflex angle $\angle BOD$

$$\angle BOD = 360^\circ - 130^\circ = 230^\circ$$

Thus, $\angle BCD = \frac{1}{2}\angle BOD$

$$= \frac{1}{2} \times 230^\circ$$
$$= 115^\circ$$

10. Guangzhou circle is a landmark located in the chinese city Guangzhou. This doughnut-shaped building is the headquarters of the Hongda Xingye group. One day a group of students with class teacher visited the place and took a photograph of that building. The teacher marked some points on the photograph and joined them as shown below.



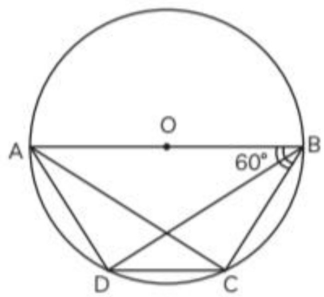
If BD is the longest chord of the circular building, then which of the following will be the diameter of the building?

(a) AB **(b) BD**
(c) AD **(d) None of these**

Ans. *(b) BD*

Explanation: Diameter of a circle is the longest chord of the circle. Therefore, the diameter of the circle will be BD.

11. **In the figure shown below, O is the centre of the circle. If ∠ABC = 60°, then ∠BDC is:**



(a) 30° **(b) 45°**
(c) 25° **(d) 25°**

Ans. *(a) 30°*

Explanation: In ΔABC,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$ [Angle Sum Property]

$\Rightarrow$ $\angle 60^\circ + \angle BAC + 90^\circ = 180^\circ$ [∠ACB is the angle in a semicircle]

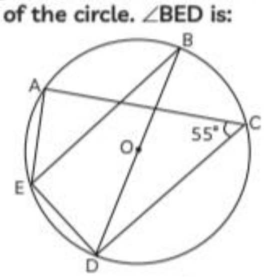
$\Rightarrow$ $\angle BAC = 180^\circ - 150^\circ$

$\Rightarrow$ $\angle BAC = 30^\circ$

Now, for chord BC, $\angle BAC = \angle BDC$ [Angles made by the same segment]

Therefore, $\angle BDC = 30^\circ$

12. **In the figure, ∠ACD = 55° and BD is the diameter of the circle. ∠BED is:**



(a) 35° **(b) 55°**
(c) 25° **(d) Cannot be determined**

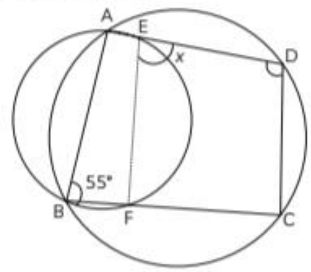
Ans. *(d) Cannot be determined*

Explanation: Here, the angle made by arc ACD is 55° and we have to find ∠BED made by the arc BED. But there is no relation between arcs ACD and BED.

Caution

Students will be confused with ∠ACD = ∠EBD, but it's not ∠ACD ≠ ∠EBD because ∠EBD and is ∠ACD made on different segments.

13. **If EF || DC and ABCD is a cyclic quadrilateral, then the value of *x* is:**



(a) 55° **(b) 125°**
(c) 45° **(d) 90°**

Ans. *(a) 55°*

Explanation: As ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary.

$\Rightarrow$ $\angle B + \angle D = 180^\circ$

$\Rightarrow$ $55^\circ + \angle D = 180^\circ$

$\Rightarrow$ $\angle D = 180^\circ - 55^\circ$

$\Rightarrow$ $\angle D = 125^\circ$

Now, EF || CD

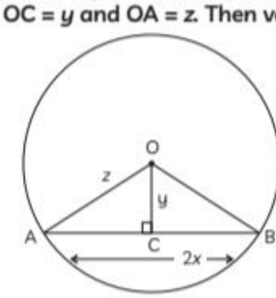
$\Rightarrow$ $x + \angle D = 180^\circ$ [Co-interior angles]

$x + 125^\circ = 180^\circ$

$x = 180^\circ - 125^\circ$

$x = 55^\circ$

14. **In the given figure, *x*, *y* and *z* are three consecutive integers and OC ⊥ AB at C. If AB = 2*x*, OC = *y* and OA = *z*. Then value of *x* is:**



(a) 3 **(b) 4**
(c) 5 **(d) 8**

Ans. *(a) 3*

Explanation: Here, *x*, *y* and *z* are three consecutive integers.

Let $x = a$, then $y = a + 1$ and $z = a + 2$

Now, OC ⊥ AB,

Therefore, OC bisects chord AB.

or, $AC = \frac{1}{2}\ AB = \frac{1}{2} \times 2x = x$

or, $AC = a$

And, $OA = a + 2$, $OC = a + 1$

In ΔOAC,

$$OA^2 = OC^2 + AC^2$$

[By Pythagoras Theorem]

$\Rightarrow \quad (a + 2)^2 = (a + 1)^2 + a^2$

$\Rightarrow \quad a^2 + 4 + 4a = a^2 + 2a + 1 + a^2$

$\Rightarrow \quad a^2 - 2a - 3 = 0$

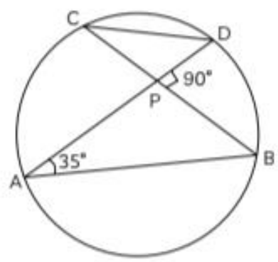
$\Rightarrow (a - 3)(a + 1) = 0 \Rightarrow a = 3$ and $a = -1$

Since, $a \neq -1$ for the length of a side.

Hence, $a = 3$

So, we have, $x = a = 3$

15. Renu draw a figure as shown below. If $\angle BAP = 35^\circ$ then $\angle CDP$ is:



(a) 55° **(b) 35°**
(c) 25° **(d) 125°**

Ans. *(a) 55°*

Explanation: $\angle DCB = \angle BAD$

[Angles made on the same segment]

$\angle DCB = 35^\circ$

Now, in ΔCDP

$\angle C + \angle D + \angle P = 180^\circ$

[Angle sum property]

$35^\circ + \angle D + 90^\circ = 180^\circ$

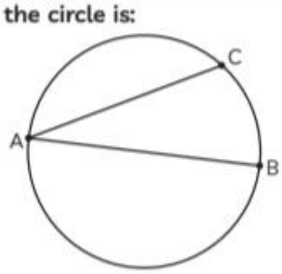
[$\angle DPB = \angle DPC = 90^\circ$]

$\angle D = 180^\circ - 35^\circ - 90^\circ$

$\angle D = 55^\circ$

$\angle CDP = 55^\circ$

16. In the given figure, AB is a diameter of a circle and AC is a chord. If AB = 34 cm, AC = 30 cm, the distance of AC from the centre of the circle is:

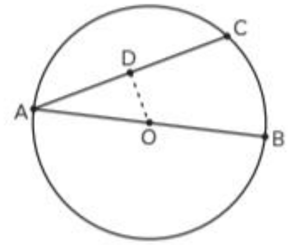


(a) 17 cm **(b) 15 cm**
(c) 4 cm **(d) 8 cm**

[British Council 2022]

Ans. *(d) 8 cm*

Explanation: Draw $OD \perp AC$

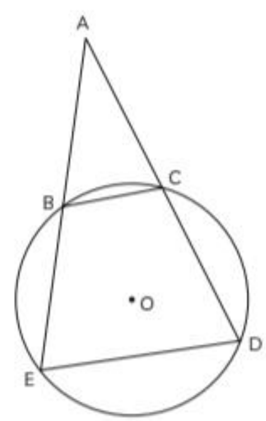


[Perpendicular from the centre bisects the chord]

$AD = 15$ cm, $AO = 17$ cm

$OD = \sqrt{17^2 - 15^2} = 8$ cm

17. In the given figure, AC = 8 cm, BC || DE and EB and DC when produced meet at A. The length of AB is:



(a) 4 **(b) 8**
(c) 7 **(d) 5**

Ans. *(b) 8*

Explanation: Given that BC || ED so,

$\angle ABC = \angle AED$ and $\angle ACB = \angle ADE$

[Corresponding angles]

But, BCDE is a cyclic quadrilateral.

Therefore, $\angle ABC = \angle ADE$ and $\angle ACB = \angle AED$

$\Rightarrow \quad \angle ABC = \angle ACB$

Thus, in ΔABC,

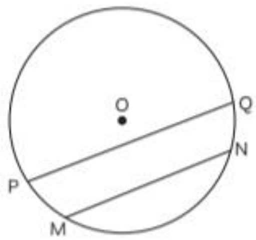
$\angle ABC = \angle ACB$

$AB = AC$

[Sides of isosceles triangle]

Hence, $\quad AB = 8$ cm

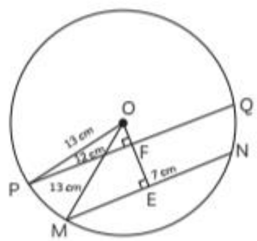
18. Given below is a circle with centre O and radius 13 cm. PQ and MN are two chords of length 24 cm and x cm respectively. The distance between the chord is 7 cm. What is the value of x?



(a) 10 **(b) 5**
(c) 11 **(d) 6** **[Diksha]**

Ans. *(a) 10*

Explanation:



Given: OP = OM = 13 cm

FE = 7 cm

The perpendicular drawn from the centre bisects the chord.

$$PF = \frac{1}{2} PQ$$

$$PF = \frac{1}{2} \times 24$$

$$PF = 12 \text{ cm}$$

In ΔOPF,

$$OP^2 = PF^2 + OF^2$$

[By Pythagoras Theorem]

$$13^2 = 12^2 + OF^2$$

$$OF^2 = 169 - 144$$

$$OF^2 = 25$$

$$OF = \sqrt{25} = 5 \text{ cm}$$

Now, $OE = OF + FE$

$= (5 + 7)$ cm

$OE = 12$ cm

Now, in ΔOME,

$$OM^2 = OE^2 + ME^2$$

$$13^2 = 12^2 + ME^2$$

$$169 = 144 + ME^2$$

$$ME^2 = 169 - 144$$

$$ME^2 = 25$$

$$ME = \sqrt{25}$$

$$ME = 5 \text{ cm}$$

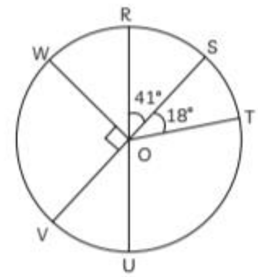
$$MN = 2ME$$

$$MN = 2 \times 5 \text{ cm}$$

$$MN = 10 \text{ cm}$$

Hence, $x = MN = 10$ cm

19. In the figure shown below, VS and UR are the diameters of the circle with centre O. Which among the following minor arcs has the greatest measure at O?



(a) WT **(b) VR**
(c) UW **(d) TU**

Ans. *(b) VR*

Explanation: $\angle WOV + \angle WOR + \angle ROS = 180°$

[VS is diameter]

$90° + \angle WOR + 41° = 180°$ [WO $\perp$ VS]

$\angle WOR = 180° - 90° - 41°$

$\angle WOR = 49°$

Now,

m$\widehat{WT}$ makes an angle $49° + 41° + 18° = 108°$

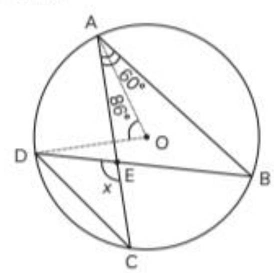
m$\widehat{VR}$ makes an angle $90° + 49° = 139°$

m$\widehat{UW}$ makes an angle $180° - 49° = 131°$

m$\widehat{TU}$ makes an angle $180° - 41° - 18° = 121°$

So, minor arc VR has the greatest measure at O.

20. In the figure shown below, O is the centre of the circle, $\angle BAC = 60°$ and $\angle AOD = 86°$. The value of x is:



(a) 66° **(b) 77°**
(c) 43° **(d) 107°**

Ans. *(b) 77°*

Explanation: Given,

$\angle BAC = 60°$ and $\angle AOD = 86°$

Here, $\angle BAC = \angle BDC$

[arc BC is Common]

Therefore, $\angle BDC = 60°$

Now, O is the centre of the circle,

and $\angle AOD = 86^\circ$

So, $\angle AOD = 2\angle ACD$ [arc AD is Common]

Or, $\angle ACD = \frac{1}{2}\angle AOD$

$= \frac{1}{2} \times 86^\circ$

$= 43^\circ$

Now, in ΔDEC,

$\angle EDC + \angle ECD + x = 180^\circ$ [Angle sum property]

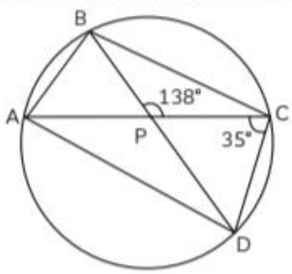
Since, $\angle EDC$ and $\angle BDC$ are equal and also $\angle ECD$ and $\angle ACD$ are equal.

$\Rightarrow 60^\circ + 43^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 103^\circ$

$\Rightarrow x = 77^\circ$

21. ABCD is a cyclic quadrilateral. The diagonals BD and AC intersect each other at point P. If the measure of $\angle BPC = 138^\circ$ and $\angle PCD = 35^\circ$, then the value of double of $\angle BAC$ is:



(a) 133° **(b) 203°**
(c) 103° **(d) 206°**

Ans. *(d) 206°*

Explanation: Angle made by the same segments are equal

Therefore, $\angle ACD = \angle ABD$

$\angle ABD = 35^\circ$

Now, on diagonal AC,

$\angle APB + \angle BPC = 180^\circ$ [Linear angle property]

$\angle APB = 180^\circ - 138^\circ$

$\angle APB = 42^\circ$

Now, in ΔAPB,

$\angle A + \angle P + \angle B = 180^\circ$ [Angle sum property]

$\angle A = 180^\circ - 35^\circ - 42^\circ$

$\angle A = 103^\circ$

$\therefore \angle BAC = 103^\circ$

According to the question,

$2\angle BAC = 2 \times 103^\circ$

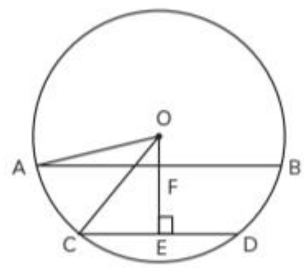
$= 206^\circ$

22. In a circle with centre O and diameter 50 cm. AB and CD are two parallel chords of length 30 cm and *x* cm, respectively and the chords are on the same side of the centre O. The distance between the chord is 4 cm. What is the value of chord CD?

(a) 7 cm **(b) 28 cm**
(c) 14 cm **(d) 40 cm**

Ans. (c) *14 cm*

Explanation: Here, first we draw the figure as shown below.



Now, diameter = 50 cm

So, radius, OA = OC = 25 cm

Now, in ΔOAF,

$OA^2 = AF^2 + OF^2$

$25^2 = 15^2 + OF^2$

$OF = \sqrt{625 - 225} = 20$ cm

So, $OE = OF + EF = 20 + 4 = 24$ cm

Now, in ΔOCE,

$OC^2 = OE^2 + CE^2$

$25^2 = 24^2 + CE^2$

$CE^2 = 625 - 576$

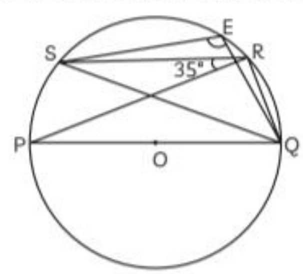
$CE^2 = 49$

$CE = \sqrt{49}$

$CE = 7$

$CD = 2CE = 2 \times 7 = 14$ cm

23. PQ is the longest chord of the circle with centre O. The points R and S are on the opposite sides of the longest chord PQ, such that $\angle SRP = 35^\circ$. The point E is on the minor arc SQ. What is the measure of $\angle SEQ$?



(a) 115° **(b) 125°**
(c) 55° **(b) 90°**

Ans. *(b) 125°*

Explanation: The longest chord of the circle is the diameter of the circle.

So, diameter is PQ

We know that

Angle in a semicircle is 90°

Therefore, ∠PRQ = 90°

And, ∠SRQ = 90° + ∠SRP

= 90° + 35°

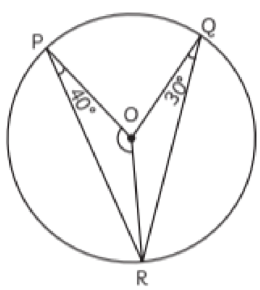
∠SRQ = 125°

Therefore, ∠SRQ = ∠SEQ

[Angle made by the same segment are equal]

Hence, ∠SEQ = 125°

24. Point P and Q are on the circle with centre O. Point R is on the major arc PQ. If ∠OPR = 40° and ∠OQR = 30°, then what is the measure of the angle subtended by the minor arc PQ at the centre?



(a) 110° **(b)** 70°
(c) 140° **(d)** 40°

Ans. *(c) 140°*

Explanation:

Here, OP = OQ = OR [Radius of the circle]

So, ∠OPR = ∠ORP = 40°

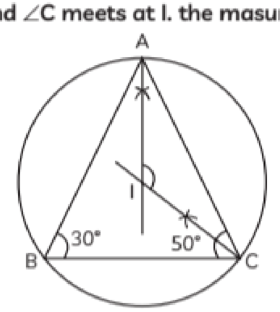
Similarly, ∠OQR = ∠ORQ = 30°

Therefore, ∠PRQ = 30° + 40° = 70°

and, ∠POQ = 2 × 70° = 140°

[The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.]

25. A triangle ABC is inscribed in a circle with ∠B = 30° and ∠C = 50°. The angle bisectors of ∠A and ∠C meets at I. the masure of ∠AIC is:



(a) 105° **(b)** 100°
(c) 75° **(d)** 120°

Ans. *(a) 105°*

Explanation: In ΔABC,

∠A + ∠B + ∠C = 180°

[Angle sum property]

∠A + 30° + 50° = 180°

∠A = 180° – 30° – 50° = 100°

Since, we know that angle bisector divides the angle in equal parts.

Therefore, $\angle ACI = \frac{\angle C}{2} = \frac{50°}{2} = 25°$

$\angle CAI = \frac{100}{2} = 50°$

In ΔAIC,

∠ACI + ∠CAI + ∠AIC = 180°

25° + 50° + ∠AIC = 180°

∠AIC = 180° – 25° – 50°

∠AIC = 105°

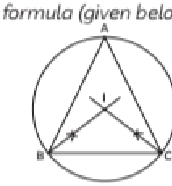
Caution

➥ *Students should remember that the incentre of the triangle can also be found by formula (given below)*

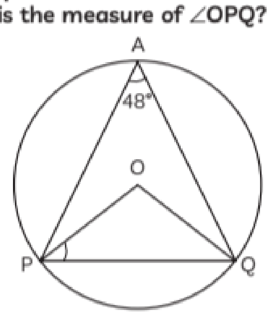
∠BIC = 90° + A

$\angle AIC = 90° + \frac{\angle B}{2}$

$\angle AIB = 90° + \frac{\angle C}{2}$



26. PQ is a chord of a circle with centre O and A is any point on the circle. If ∠PAQ = 48°, then what is the measure of ∠OPQ?



(a) 150° **(b)** 32°
(c) 42° **(d)** 48°

Ans. *(c) 42°*

Explanation: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Therefore, ∠POQ = 2∠PAQ

= 2 × 48°

∠POQ = 96°

Since, OP = OQ

Therefore, let ∠OPQ = ∠OQP = $x°$

In ΔOPQ,

∠POQ + ∠OPQ + ∠OQP = 180°

[Angle sum property]

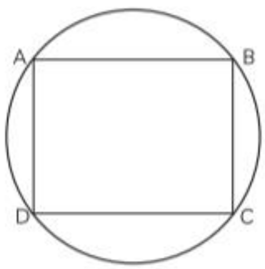
$96° + x + x = 180°$

$96° + 2x = 180°$

$x = \frac{180° - 96}{2} = 42°$

Hence, $\angle OPQ = 42°$

27. In the figure shown below, ABCD is a cyclic quadrilateral in which ∠C is 3 times of ∠A and ∠B is 5 times of ∠D. What is the value of (3∠A – ∠D)?



(a) 120° **(b) 60°**
(c) 90° **(d) 105°**

Ans. *(d) 105°*

Explanation:

Let, $\angle A = x$

$\angle C = 3\angle A$

$\angle C = 3x$

Also, $\angle D = y$

$\angle B = 5\angle D = 5 \times y = 5y$

Now, in a cyclic quadrilateral, opposite angles are supplementary.

$\angle A + \angle C = 180°$

$x + 3x = 180°$

$4x = 180°$

$x° = \frac{180}{4} = 45°$

$\angle A = 45°$

Also, $\angle D + \angle B = 180°$

$y + 5y = 180°$

$6y = 180°$

$y = \frac{180}{6} = 30°$

$\angle D = y = 30°$

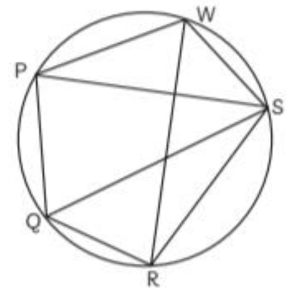
Therefore, $3\angle A - \angle D = 3 \times \angle A - \angle D$

$= 3 \times 45° - 30°$

$= 135° - 30°$

$= 105°$

28. P, Q, R, S, and W are concyclic. If ∠PWR = 50° and ∠PSQ = 30°, then what is the value of ∠QSR?



(a) 15 **(b) 20**
(c) 32 **(d) 10**

Ans. *(b) 20*

Explanation:

$\angle PWR = 50°$ [Given]

$\angle PWR = \angle PSR$

[Angles made by the same segment are equal]

$\angle PSR = 50°$

$\angle PSR = \angle PSQ + \angle QSR$

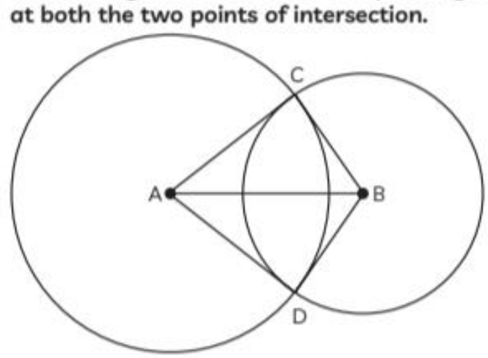
$50° = 30° + \angle QSR$

$\angle QSR = 50° - 30°$

$\angle QSR = 20°$

True and False

29. The lines joining the centres of any two intersecting circles subtend unequal angles at both the two points of intersection.



Ans. *False*

Explanation: The lines joining the centres of any two intersecting circles subtend equal angles at both the points of intersection.

In ΔACB and ΔADB,

$AB = AB$ [Common]

$AC = AD$ [Equal radius with centre A]

$BC = BD$ [Equal radius with centre B]

So, by SSS rule of congruence,

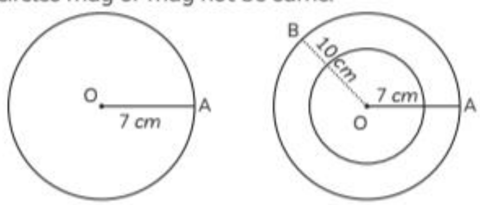
$\Delta ACB \cong \Delta ADB$

$\Rightarrow \angle ACB = \angle ADB$

30. Two circles having same centre are always equal in area.

Ans. *False*

Explanation: The circles having same centre are called concentric circle. Radii of concentric circles may or may not be same.



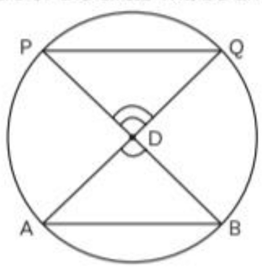
The area of a circle depends upon its radius.

So, two circles having same centre are not always equal in area.

31. The angles subtended by two unequal chords of a circle could be equal.

Ans. *False*

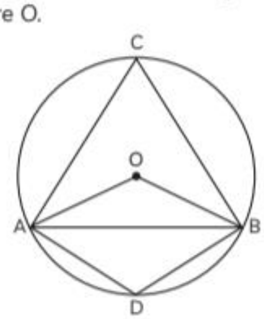
Explanation: Only equal chords of a circle subtend equal angles at its centre. If the chords of the circle are unequal, then subtended angles at the centre of the circle will be unequal.



32. If the chord of a circle is equal to its radius, then the angles subtended by the chord at a point on the minor arc and at a point on the major arc are supplementary.

Ans. *True*

Explanation: Let us draw the figure of a circle with centre O.



Here, $\angle ADB$ and $\angle ACB$ are two angles on minor arc and major arc with common chord AB.

Now, in ΔOAB,

OA = OB = AB [Radius and chord are equal]

So, ΔOAB is an equilateral triangle.

Hence, $\angle AOB = 60^\circ$

And, $\angle ACB = \frac{1}{2}\angle AOB$

[Angle subtended by segment at the circumference is half of the angle subtended by the same segment at the centre]

$\Rightarrow \angle ACB = \frac{1}{2} \times 60^\circ$

$\Rightarrow \angle ACB = 30^\circ$

Now, reflex angle, $\angle AOB$

$= 360^\circ - \angle AOB$

$= 360^\circ - 60^\circ$

$= 300^\circ$

So, $\angle ADB = \frac{1}{2} \times \text{Reflex } \angle AOB$

$= \frac{1}{2} \times 300^\circ = 150^\circ$

So, we have, $\angle ACB + \angle ADB$

$= 30^\circ + 150^\circ$

$= 180^\circ$

33. An equilateral triangle can be inscribed in a semicircle with its one side as the diameter of the semicircle.

Ans. *False*

Explanation: It is not possible to inscribe an equilateral triangle in a semicircle with one side as diameter of the semicircle because all the sides of the equilateral triangle are equal and form congruent angles, also we know that the angle in a semicircle is 90°.

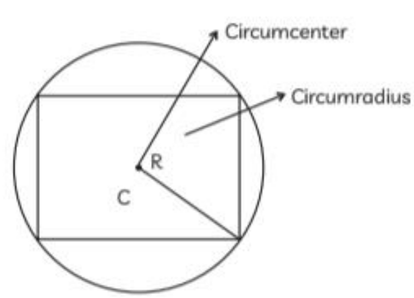
Hence, the answer is false.

Fill in the Blanks

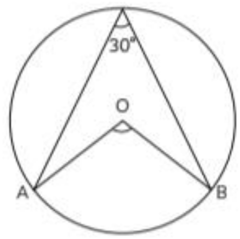
34. A cyclic quadrilateral is a quadrilateral inscribed in a circle. The centre of this circle is known as .... of the circle.

Ans. *circumcenter*

Explanation: As the circle circumscribing the quadrilateral, so its centre is called as circumcentre.



35. **In a circle with centre O, if the angle subtended by minor arc AB at the boundary of the circle is 30°, then the measure of ∠AOB will be ....**



Ans. *60°*

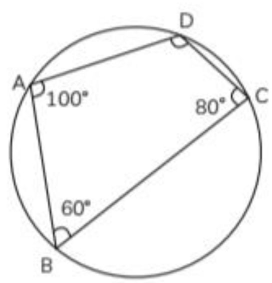
Explanation: As we know the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

36. **If the length of an arc is more than the length of the arc of the semicircle, then it is called ....**

Ans. *major arc*

Explanation: Length of major arc of a circle is more than the length of arc of the semicircle and the length of minor arc of the circle is less than the length of arc of the semicircle.

37. **If ABCD is a cyclic quadrilateral such that ∠A = 100°, ∠B = 60°, ∠C = 80°, then ∠D will be equal to ....**



Ans. 120°

Explanation: Opposite angles of a cyclic quadrilateral are supplementary.

So, ∠A + ∠C = 180°

And, ∠B + ∠D = 180°

∠D = 180° – 60° = 120°

38. **ABCD is a cyclic quadrilateral. If ∠A is 30°, then ∠C = ...**

Ans. ∠150°

Explanation: The sum of either pair of the opposite angles of a cyclic quadrilateral is 180°.

∠A + ∠C = 180°

30° + ∠C = 180°

∠C = 180° – 30°

∠C = 150°

Assertion and Reason (A-R)

Direction for questions 39 to 43: In question number 39 to 43, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

39. **Assertion (A): The chord, which divides the circle into two equal parts, passes through the centre.**

Reason (R): The chord of a circle always, divides the circle into two equal parts.

Ans. *(c) Assertion (A) is true but reason (R) is false.*

Explanation: Diameter is the longest chord of the circle which divides the circle into two equal parts. Only the longest chord of the circle divides the circle into two equal parts.

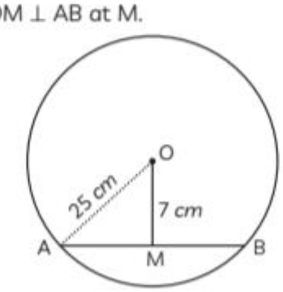
40. **Assertion (A): The length of a chord which is at a distance of 7 cm from the centre of a circle of radius 25 cm, is 48 cm.**

Reason (R): The perpendicular from the centre of a circle to a chord bisects the chord.

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation: Let AB be the chord of the circle with centre O and radius, OA = 25 cm.

Draw, OM ⊥ AB at M.



Here, OM = 7 cm [Given]

In right-angled triangle OAM,

$OA^2 = OM^2 + AM^2$

$\Rightarrow AM^2 = 25^2 - 7^2$

$= 625 - 49$

$= 576$

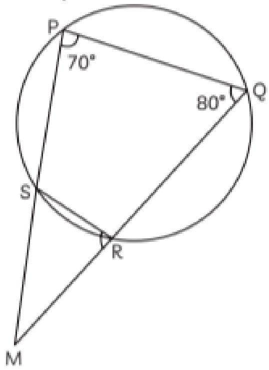
$\Rightarrow AM = \sqrt{576} = 24$ cm

Since, the perpendicular from the centre to a chord of the circle bisects the chord.

So, AB = 2 AM

$\Rightarrow$ AB = 2 × 24 = 48 cm.

41. Assertion (A): PQRS is a cyclic quadrilateral PS and QR produced to meet in M. If ∠QPM = 70°. and ∠PQM = 80°, then ∠SRM will be equal to 80°.



Reason (R): Exterior angle in a cyclic quadrilateral is equal to opposite interior angle.

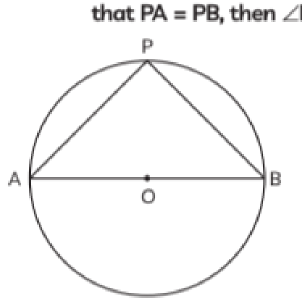
Ans. *(d) Assertion (A) is false but reason (R) is true.*

Explanation:

Exterior angle in a cyclic quadrilateral is equal to opposite interior angle.

So, ∠SRM = ∠QPS = ∠QPM = 70°.

42. Assertion (A): AB is the diameter of a circle with centre O. P is a point on the circumference of the circle. Such that PA = PB, then ∠PAB = 45°



Reason (R): Angle in a semicircle is a right angle.

Ans. *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation:

In ΔAPB,

PA = PB [Given]

So, ∠PAB = ∠PBA [Angle of an isosceles triangle]

= x (say)

Now, angle in a semicircle is a right angle.

∠APB = 90° [AB is diameter]

So, ∠PAB + ∠PBA + ∠APB = 180°

$$x + x + 90° = 180°$$

$$x = \frac{90°}{2} = 45°$$

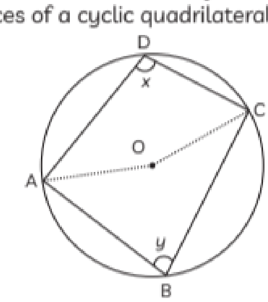
Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

43. Assertion (A): If the diagonals of a cyclic quadrilateral bisect each other, then it is a parallelogram.

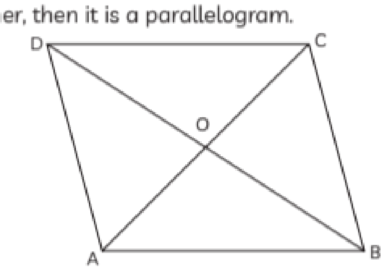
Reason (R): All the vertices of the cyclic quadrilateral lie on the circle.

Ans. *(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation: If any four points on the circumference of a circle are joined, they form the vertices of a cyclic quadrilateral.



If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



Given that, OA = OC and OB = OD

Now, ΔDOC and ΔBOA

OD = OB

OC = OA

∠DOC = ∠BOA [Vertically opposite angles]

[By SAS congruence rule]

ΔDOC ≅ ΔBOA

$\Rightarrow$ ∠ODC = ∠OBA [Alternate angles]

$\Rightarrow$ CD || AB

Similarly, AD || BC

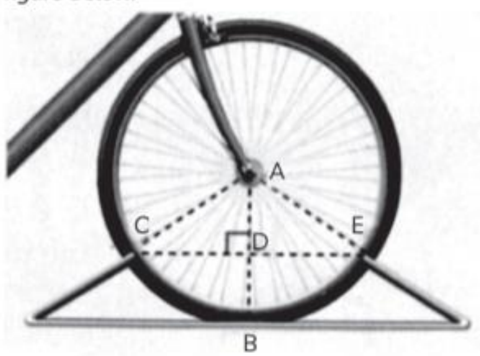
So, ABCD is a parallelogram.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

44. *Sushant and Karan, in order to maintain a healthy life, planned morning cycling; one day they started racing and Karan left Sushant behind and meanwhile Sushant's cycle wheel punctured. Now sitting beside the road he started observing the wheel. His bicycle has a wheel 26 cm in diameter as shown in the figure below.*



(A) What is the length of AC?

(a) 10 cm (b) 13 cm
(c) 15 cm (d) 20 cm

(B) What is the length of AD, if BD is 7 cm?

(a) 1 cm (b) 8 cm
(c) 9 cm (d) 6 cm

(C) What is the length of CD to the nearest tenths?

(a) 11.5 cm (b) 10 cm
(c) 9 cm (d) 12 cm

(D) What is the length of the top of the bike stand, CE?

(a) 40 cm (b) 24 cm
(c) 23 cm (d) 36 cm

(E) Find the distance covered by the wheel in one revolution.

(a) 80 cm (b) 79 cm
(c) 98.18 cm (d) 81.71 cm

Ans. (A) *(b) 13 cm*

Explanation: Diameter = 26 cm

Diameter = 2 × radius

So, radius $(r) = \frac{26}{2} = 13$ cm

Or, AC = 13 cm

(B) *(d) 6 cm*

Explanation: Here, radius, AC = AB = AE

Now, AB = AD + BD

$\Rightarrow$ 13 = AD + 7

$\Rightarrow$ AD = (13 − 7) cm

$\Rightarrow$ AD = 6 cm

(C) *(a) 11.5 cm*

Explanation: Apply Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$
$$CD^2 = AC^2 - AD^2$$
$$CD^2 = 13^2 - 6^2$$
$$= 169 - 36$$
$$CD = \sqrt{133}$$
$$CD = 11.5 \text{ cm}$$

(D) *(c) 23 cm*

Explanation: If the radius of circle is perpendicular to a chord, then it bisects the chord.

So, CD = ED

Hence, CE = 2CD

= 2 × 11.5

= 23 cm (Approx)

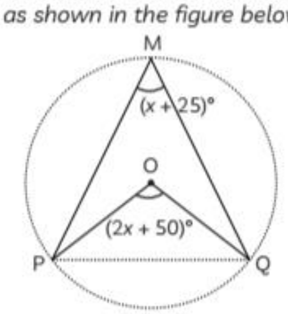
(E) *(d) 81.71 cm*

Explanation: The distance covered by the wheel in 1 revolution is equal to the measure of its circumference.

Circumference = $2\pi r$

$$= 2 \times \frac{22}{7} \times 13$$
$$= 81.71 \text{ cm}$$

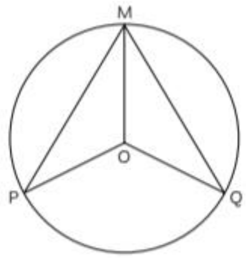
45. *During a practical activity in maths lab, students were using circular geoboard. The angle subtended by an arc PQ at the centre O is $(2x + 50)°$. Sunita calculated angle PMQ as $(x + 25)°$ as shown in the figure below.*



(A) Is Sunita's answer correct? Justify it.

(B) Suppose, the length of the chord PQ is equal to the radius of the circle, then what is the value of *x*?

(C) MP and MQ are two equal chords of the circle. Prove that MO bisects ∠PMQ.



Ans. (A) Yes

Here, $\angle POQ = 2\angle PMQ$

Or, $\angle PMQ = \frac{1}{2}\angle POQ$

[Angle subtended at the centre of circle is twice the angle subtended at its circumference]

Therefore, $\angle PMQ = \frac{1}{2}\angle POQ = \frac{1}{2}\times(2x+50)^\circ$

$= \frac{1}{2}\times 2\times(x+25)^\circ$

$= x + 25^\circ$

(B) Given that PQ is equal to the radius of the circle.

Or, $PQ = OP = OQ$

Thus, ΔPOQ is an equilateral triangle.

So, $\angle POQ = 60^\circ$

Hence, $2x + 50^\circ = 60^\circ$

$\Rightarrow 2x = 60^\circ - 50^\circ$

$\Rightarrow x = \frac{10^\circ}{2} = 5^\circ$

(C) Join OP and OQ

In ΔOPM and ΔOQM

$OP = OQ$ [Radii of the circle]

$OM = OM$ [Common]

$MP = MQ$ [Given]

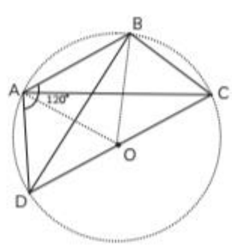
$\Delta OAB \cong \Delta OCB$

[By SSS Congruence Rule]

$\angle PMO = \angle QMO$

Hence, MO bisects PMQ.

46. *Monika's mathematics teacher drew a circle on the board and put the centre of the circle as O. Shelly drew a line DC parallel to AB and joined ABCD to make the cyclic quadrilateral ABCD.*



(A) If ABCD is a cyclic quadrilateral then ∠BCD is:

(a) 120° (b) 90°

(c) 60° (d) 30°

(B) If an ant starts from O and moves to B making a line of 6 cm, then the length of OA is:

(a) 6 cm (b) 11 cm

(c) 9 cm (d) 10 cm

(C) What is the measure ∠COD?

(a) 100° (b) 60°

(c) 40° (d) 180°

(D) If BC = 5 cm and BD = 12 cm, then the length of DC is:

(a) 10 cm (b) 13 cm

(c) 9 cm (d) 7 cm

(E) What is the measure of ∠ACD if AB = AD?

(a) 60° (b) 100°

(c) 30° (d) 80°

Ans. (A) *(c) 60°*

Explanation: As ABCD is a cyclic quadrilateral and opposite angles of a cyclic quadrilateral are supplementary.

Therefore, $\angle A + \angle C = 180^\circ$

$120^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 120^\circ$

$\angle C = 60^\circ$

(B) *(a) 6 cm*

Explanation: Here, OA, OB are the lines connecting the centre to the circumference. So, these are radii of the circle with centre O. Thus, OB and OA are of the same length.

$OB = OA = 6$ cm

(C) *(d) 180°*

Explanation: Since, COD is a stright line and also the diameter of the circle with centre be O.

Hence, $\angle COD = 180^\circ$

(D) *(b) 13 cm*

Explanation: Angle made in a semicircle is 90°.

In ΔBDC

$\angle B = 90^\circ$

$DC^2 = DB^2 + BC^2$

[Pythagoras Theorem]

$DC^2 = 12^2 + 5^2$

$DC^2 = 169$

$DC = 13$ cm

(E) *(c) 30°*

Explanation: In ΔABD,

$\angle ABD + \angle ADB + \angle BAD = 180°$

[Angle sum property]

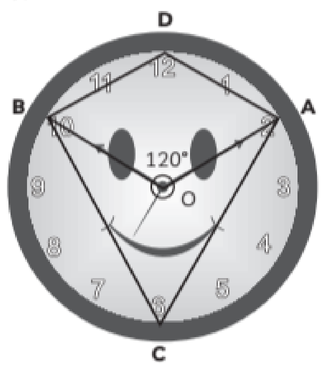
Here, $AB = AD$

So, $\angle ABD = \angle ADB = x$ (Say)

$\Rightarrow$ $x + x + 120° = 180°$

$\Rightarrow$ $x = \dfrac{60°}{2} = 30°$

47. *Shranya brought a beautiful clock that has a circular dial. She joined the digits 2 and 10 with 6 and the digits 10 and 2 with the centre of circle O.*



(A) If $\angle ACO = 30°$, then find $\angle AOC$.

(B) In quadrilateral AOBD, if $\angle AOB = 120°$, then find $\angle ADB$.

(C) Find the type of angle, $\angle ADB$ will make?

Ans. (A) In ΔOAC,

$\angle OCA = \angle OAC = 30°$[OA = OC]

Therefore,

$\angle OCA + \angle OAC + \angle AOC = 180°$

[Angle sum property]

$30° + 30° + \angle AOC = 180°$

$\angle AOC = 180° - 60°$

$\angle AOC = 120°$

(B) In quadrilateral AOBD,

$\angle AOB = \angle 120°$ [Given]

Reflex $\angle AOB = 360° - 120° = 240°$

Now,

Reflex $\angle AOB = 2 \times \angle ADB$

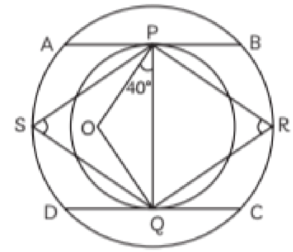
Or, $\angle ADB = \dfrac{1}{2} \times \text{Reflex } \angle AOB$

$= \dfrac{1}{2} \times 240°$

$= 120°$

(C) Since, $\angle ADB = 120°$, *i.e.,* greater than 90° but less than 180°. So $\angle ADB$ is an obtuse angle.

48. *For an interior design project, Raj, being an architect, drew a circular design of a building. In this picture, two circles with centre O have been drawn.*



(A) Find the value of $\angle OQP$.

(B) Find the value of $\angle POQ$.

(C) Find the value of x and y.

Ans. (A) In ΔOPQ,

QP = OQ = Radius of inner circle with centre O.

Hence, $\angle OPQ = \angle PQO = 40°$

[Angle opposite to equal sides are equal]

(B) In ΔOPQ,

$\angle OPQ + \angle PQO + \angle POQ = 180°$

[Angle sum property]

$40° + 40° + \angle POQ = 180°$

$\angle POQ = 180° - 80°$

$\angle POQ = 100°$

(C) Reflex $\angle POQ = 360° - 100°$

$\angle POQ = 260°$

$x = \dfrac{1}{2} \times \text{Reflex } \angle POQ$

[Angle made by an arc on the centre is double to that of circumference]

$x = \dfrac{1}{2} \times 260°$

$x = 130°$

$\angle POQ = 100°$

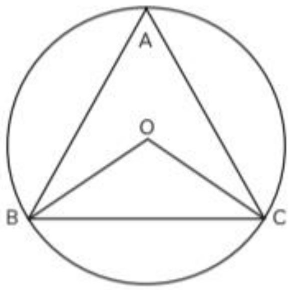
$y = \dfrac{1}{2} \angle POQ$

[Angle made by an arc on the centre is double to that a circumference]

$y = \dfrac{1}{2} \times 100°$

$y = 50°$

49. *Three STD booths are placed at A, B and C in the figure and these are operated by handicapped persons. These three booth are equidistant from each other as shown in the figure.*



(A) Find the measure of ∠BAC.

(B) Find the measure of ∠BOC.

(C) If AB = 8 cm, find the value of (BC + CA).

[British Council 2022]

Ans. (A) In the figure, AB = BC = CA.

∴ ΔABC is an equilateral triangle.

⇒ ∠ABC = ∠ACB = ∠BAC = 60°

(B) It is given that OB is the bisector of ∠ABC.

$$\therefore \quad \angle ABO = \angle OBC = \frac{1}{2}\angle ABC$$

$$= \frac{1}{2} \times 60° = 30° \quad \text{...... (i)}$$

Also, OC is the bisector of ∠ACB.

$$\therefore \quad \angle ACO = \angle OCB = \frac{1}{2}\angle ACB$$

$$= \frac{1}{2} \times 60° = 30° \quad \text{...... (ii)}$$

In ΔOBC,

∠OBC + ∠OCB + ∠BOC = 180°

[Angle sum property]

⇒ 30° + 30° + ∠BOC = 180°

⇒ ∠BOC = 180° – 60° = 120°

(C) ΔABC is an equilateral triangle.

AB = BC = CA

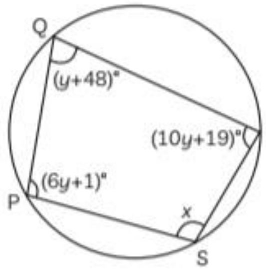
BC + CA = 8 + 8

Therefore BC + CA = 16 cm.

VERY SHORT ANSWER Type Questions (VSA)

[**1** mark]

50. In the figure shown below, what is the value of x?



Ans. ∠P + ∠R = 180°

[Cyclic quadrilateral]

$(6y + 1)° + (10y + 19)° = 180°$

$16y + 20° = 180°$

$16y = 160°$

$y = 10°$

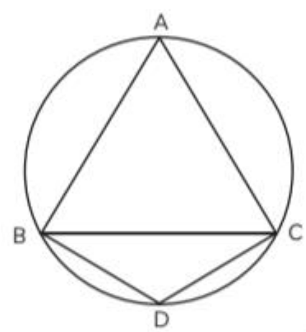
Now, ∠Q + ∠S = 180°

⇒ $(10+48)° + x = 180°$

$58° + x = 180°$

⇒ $x = 180° – 58° = 122°$

51. If ABCD is a cyclic quadrilateral and ABC is an equilateral triangle. Find ∠BDC.



[Diksha]

Ans. ΔABC is an equilateral triangle.

∴ ∠BAC = 60°

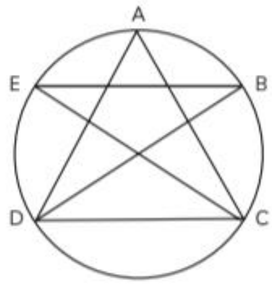
Now, ∠BCD + ∠BAC = 180°

∠BDC + 60° = 180°

∠BDC = 180° – 60°

∠BDC = 120°

52. An artist created a stained glass window as shown in the figure below. If ∠BEC = 40° and m$\widehat{AB}$ = 44°. What is ∠ADC?



Ans. ∠BEC = 40°

∠BEC = ∠BDC

[Angles with the same segment are equal]

∠BDC = 40°

m$\widehat{AB}$ = ∠ADB = 44°

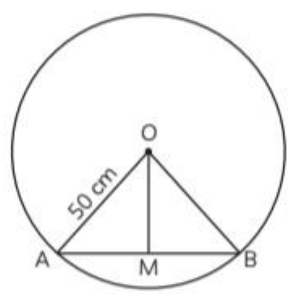
[Angles with the same segment are equal]

Hence, ∠ADC = ∠ADB + ∠BDC

= 44° + 40°

= 84°

53. In the given circle with centre O. If the radius of the circle is 50 cm, and the length of chord AB is 80 cm, then find the perpendicular distance of chord AB from the centre of circle.



Ans. Let OM ⊥ AB at M.

As we know, the perpendicular from the centre of a circle to a chord of the circle bisects the chord.

So, AM = 40 cm and AB = 80 cm

Now, in ΔOAM,

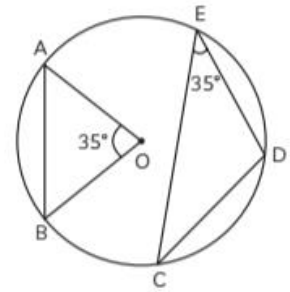
$$OM^2 = OA^2 - AM^2$$

$$= 50^2 - 40^2$$

$$= 2500 - 1600 = 900$$

⇒ $OM = \sqrt{900}$ cm = 30 cm.

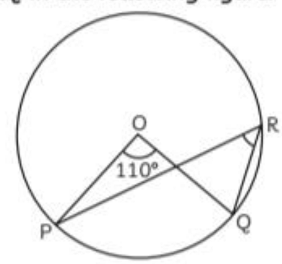
54. Sumit drawn a triangle AOB in the circle with centre O, where ∠AOB = 35°. Rishu drawn another triangle CED in the same circle, where ∠CED = 35°. Is chord AB = EC? Give reason.



Ans. No, chord AB is not equal to chord EC. Equal chords of a circle make equal angles at the centre of the circle.

Here, ∠AOC = 35° at the circumference of the circle. So, AB ≠ EC.

55. Find ∠PRQ in the following figure.



[Diksha]

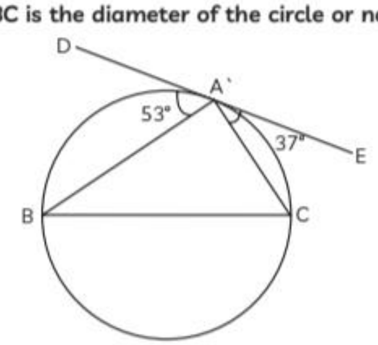
Ans. We know that angle subtended by an arc at the centre is double the angle subtended by it at the remaining part of the circle.

$$\angle PRQ = \frac{1}{2} POQ$$

$$= \frac{1}{2} \times 110°$$

$$= 55°$$

56. In the figure shown below, DE is a line passing through vertex A of the triangle ABC inside the circle with the chord BC. Check whether BC is the diameter of the circle or not.



Ans. Here, ∠DAB = 53° and ∠EAC = 37°

Now, ∠DAB + ∠BAC + ∠EAC = 180°

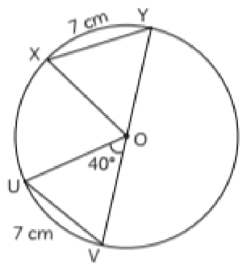
53° + ∠BAC + 37° = 180°

⇒ ∠BAC = 180° – 90° = 90°

Hence, ΔABC is a right-angled triangle.

Since, angle in a semicircle is always a right angle, so, BC is the diameter of the circle.

57. In the circle with centre O, find the measure of $\angle XOY$.

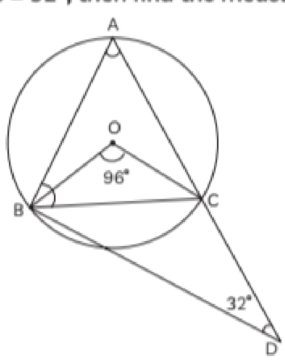


Ans. Equal chords of circle subtend equal angles at the centre of the circle.

Here, XY = UV = 7 cm and $\angle UOV = 40^\circ$

Hence, $\angle XOY = \angle UOV = 40^\circ$.

58. In the figure shown below, points A, B and C are on a circle with centre O such that $\angle BOC = 96^\circ$. If AC is produced to point D such that $\angle BDC = 32^\circ$, then find the measure of $\angle ABD$.



Ans. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\Rightarrow \quad \angle BAC = \frac{1}{2}\angle BOC$$

$$= \frac{1}{2} \times 96^\circ$$

$$\Rightarrow \quad \angle BAC = 48^\circ$$

In ΔABD,

$$\angle A + \angle B + \angle D = 180^\circ$$

[Angle sum property]

$$\Rightarrow \quad 48^\circ + \angle B + 32^\circ = 180^\circ$$

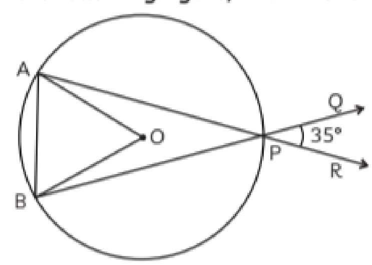
$$\Rightarrow \quad \angle B + 80^\circ = 180^\circ$$

$$\Rightarrow \quad \angle B = 180^\circ - 80^\circ$$

$$\Rightarrow \quad \angle B = 100^\circ$$

$$\Rightarrow \quad \angle ABD = 100^\circ$$

59. In the following figure, find $\angle AOB$.



[Diksha]

Ans. $\angle APB = \angle RPQ = 35^\circ$

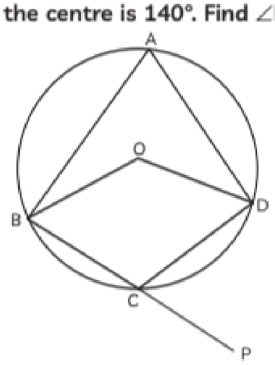
Now, $\angle AOB$ and $\angle APB$ are angles subtended by an arc AB at centre and at the remaining part of the circle.

$\therefore \quad \angle AOB = 2\angle APB = 2 \times 35^\circ = 70^\circ$

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

60. In the given figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°. Find $\angle BAD$ and $\angle DCP$.



[Diksha]

Ans. Angle BAD = $\frac{1}{2}$ Angle BOD

Angle BAD = 70°

Now, Angle BAD + Angle BCD = 180°

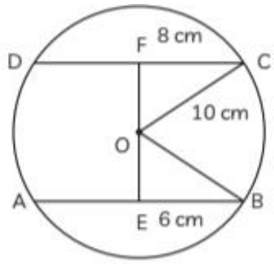
Angle BCD = 180 – 70 = 110°

Also

Angle BCD + Angle DCP = 180°

Angle DCP = 70°

61. Given below is a circle with a diameter of 20 cm. What is the distance between the chords? How much this distance will change if the chords are in the same direction of the centre?



[Diksha]

Ans. $DF = FC = \frac{1}{2} DC$

[Perpendicular from the centre bisects the chord)

$DF = FC = 8$ cm

Similarly,

$AE = EB = 6$ cm

Radius, $OC = 10$ cm

Apply Pythagoras theorem, in ΔFCO,

$$OC^2 = FO^2 + FC^2$$
$$10^2 = FO^2 + 8^2$$
$$FO^2 = 100 - 64$$
$$FO^2 = 36$$
$$FO = \sqrt{36} = 6 \text{ cm}$$

Similarly, in ΔOEB,

$$OB^2 = OE^2 + BE^2$$
$$10^2 = OE^2 + 6^2$$
$$OE^2 = 10^2 - 6^2$$
$$OE^2 = 64$$
$$OE = \sqrt{64}$$
$$OE = 8 \text{ cm}$$

Hence, the distance between the chords DC and AB, $FE = FO + OE = (6 + 8)$ cm $= 14$ cm.

Now, if the chords are in the same direction of the centre,

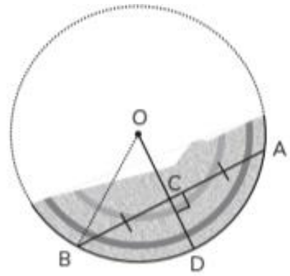
$$OE = \sqrt{OA^2 - AE^2}$$
$$= \sqrt{100 - 36} = 8 \text{ cm}$$

And $OF = \sqrt{OD^2 - DF^2} = 6$ cm

Therefore, $FE = (8 - 6)$ cm $= 2$ cm.

Clearly, $(14 - 2)$ cm $= 12$ cm distance will change if the chords are in the same direction of the centre.

62. The diagram shows a fragment of a circular plate. AB = 8 cm and CD = 2 cm. What is the diameter of a plate?



Ans. As per question $OC \perp AB$

Perpendicular is drawn from the centre bisects the chord AB, such that, $BC = AC = \frac{AB}{2} = \frac{8}{2} = 4$ cm

Radius, $OB = OD = r$

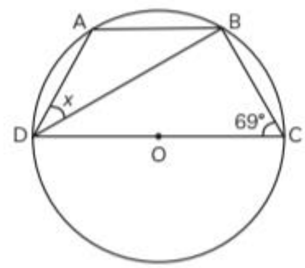
$$OD = OC + CD$$
$$OC = OD - CD$$
$$OC = r - 2$$

In ΔOBC,

$$OB^2 = OC^2 + BC^2$$
$$r^2 = (r - 2)^2 + 4^2$$
$$r^2 = r^2 + 4 - 4r + 16$$
$$4r = 20$$
$$r = 5$$

Hence, the diameter of the plate is (5×2) cm $= 10$ cm.

63. ABCD is a cyclic quadrilateral with AB || CD and DC is the diameter of the circle. If ∠BCD = 69°, then find ∠ADB.



Ans. Given that AB || CD, $\angle BCD = 69^\circ$

and DC is the diameter of the circle with centre O.

Construction: Join BD

To find: $\angle ADB$

Proof: Here, $\angle DBC = 90^\circ$ [Angle in a semicircle]

In ΔBDC,

$$\angle DCB + \angle BDC + \angle DBC = 180^\circ$$
$$69^\circ + \angle BDC + 90^\circ = 180^\circ$$
$$\angle BDC = 180^\circ - 159^\circ$$
$$= 21^\circ$$

Now, $\angle ABD = \angle BDC = 21^\circ$ [AB || DC]

And, $\angle A = 180 - \angle C$ [ABCD is a cyclic quadrilateral]

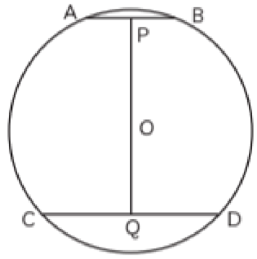
$$= 180^\circ - 69^\circ$$
$$= 111^\circ$$

Therefore, in ΔABD,

$$x = 180^\circ - \angle A - \angle ABD$$
$$= 180^\circ - 111^\circ - 21^\circ = 48^\circ$$

Hence, $\angle ADB = 48^\circ$

64. If the radius of a circle is 5 cm, AB = 6 cm and CD = 8 cm, then determine PQ.



[British Council 2022]

Ans. Since, the perpendicular from the centre of a circle to a chord bisects the chord.

Therefore, P and Q are mid-points of AB and CD respectively.

$$AP = PB = \frac{1}{2} AB = 3 \text{ cm}$$

and $CQ = QD = \frac{1}{2} CD = 4 \text{ cm}$

In right-angled ΔOAP and ΔOCQ by using Pythagoras theorem

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

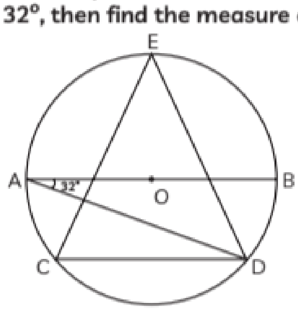
$OP^2 = 5^2 - 3^2$ $\quad$ $OQ^2 = 5^2 - 4^2$

$OP^2 = 16$ $\quad$ $= 9$

$\Rightarrow$ $OP = 4$ $\quad$ and $OQ = 3$

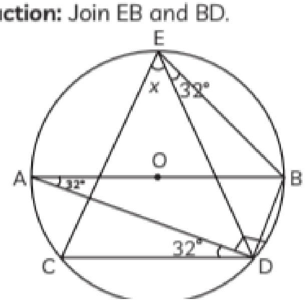
$PQ = OP + OQ = (4 + 3) = 7$ cm

65. In the given figure of a circle with centre O, chord CD is parallel to diameter AB. If $\angle BAD = 32^\circ$, then find the measure of $\angle CED$.



Ans. Given: $\angle BAD = 32^\circ$, AB || CD

Construction: Join EB and BD.



Proof: $\angle BAD = \angle ADC = 32^\circ$ [AB || CD]

$\angle ADB = 90^\circ$ [AB is diameter]

$\angle BAD = \angle BED = 32^\circ$ [Chord BD is common]

Now, in cyclic quadrilateral ECDB,

$(\angle CED + \angle BED) + (\angle ADC + \angle ADB) = 180^\circ$

[Oppostite angles of a cyclic quadrilateral]

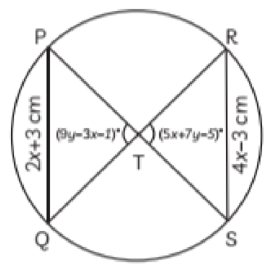
$(x + 32^\circ) + (32^\circ + 90^\circ) = 180^\circ$

$\Rightarrow$ $x = 180^\circ - 90^\circ - 64^\circ$

$\Rightarrow$ $x = 26^\circ$

$\Rightarrow$ $\angle CED = 26^\circ$

66. In the given figure, PQ and RS are equal chords of a circle with centre T. Find $\angle PTQ$.
[Diksha]



Ans. Equal chords of a circle subtend equal angles at the centre.

Here, $PQ = RS$ [Given]

So, $2x + 3 = 4x - 3$

$3 + 3 = 4x - 2x$

$2x = 6$

$x = 3$ ......(i)

So, $PQ = RS = 9$ cm

Now, $\angle PTQ = \angle RTS$

$(9y - 3x - 1)^\circ = (5x + 7y - 5)^\circ$

$(9y - 7y) = (5x + 3x - 5 + 1)$

$(2y) = (8x - 4)$

$(2y) = 8 \times 3 - 4$ [From eq. (i)]

$(2y) = 20$

$y = 10^\circ$

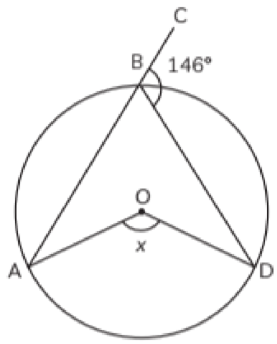
So, $\angle PTQ = 9y - 3x - 1$

$= 9 \times 10 - 3 \times 3 - 1$

$= 90 - 10$

$\angle PTQ = 80^\circ$

67. In the given figure, find the value of x, if O is the centre of the circle.



Ans. By linear pair property

$\angle CBD + \angle ABD = 180^\circ$

$146^\circ + \angle ABD = 180^\circ$

$\angle ABD = 180^\circ - 146^\circ$

$\angle ABD = 34^\circ$

Now we know,

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

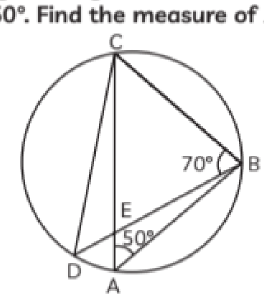
So, $\angle AOD = 2\angle ABD$

$\Rightarrow \angle AOD = 2 \times 34^\circ$

$\Rightarrow \angle AOD = 68^\circ$

$\Rightarrow x = 68^\circ$

68. **In the given figure $\angle DBC = 70^\circ$ and $\angle BAC = 50^\circ$. Find the measure of $\angle BCD$.**



[British Council 2022]

Ans. $\angle BCD = 60^\circ$

Given $\angle DBC = 70^\circ$ and $\angle BAC = 50^\circ$

$\angle CBD$ and $\angle CAD$ are angles on the same segment CD

Therefore $\angle CBD = \angle CAD$

$\angle CAD = 70^\circ$

Now $\angle BAD = \angle BAC + \angle CAD$

$\Rightarrow \angle BAD = 50^\circ + 70^\circ = 120^\circ$

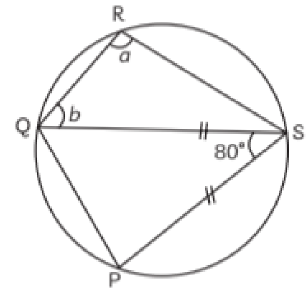
Since ABCD is a cyclic quadrilateral.

$\angle BAD + \angle BCD = 180^\circ$

$120^\circ + \angle BCD = 180^\circ$

$\angle BCD = 60^\circ$

69. **PQRS is a cyclic quadrilateral and QS = SP and QR || SP, find *a* and *b*.**



Ans. In ΔPQS,

QS = SP [Given]

Let, $\angle SPQ = \angle SQP = x$

So, $\angle SPQ + \angle SQP + \angle QSP = 180^\circ$

$x + x + 80^\circ = 180^\circ$

$2x = 180^\circ - 80^\circ$

$2x = 100^\circ$

$x = 50^\circ$

Now, given that QR || SP

So, $\angle RQS = \angle SPQ$

Or, $b = 80^\circ$

[Alternate angles]

Also, $\angle QPS = 50^\circ$

Now, PQRS is a cyclic quadrilateral

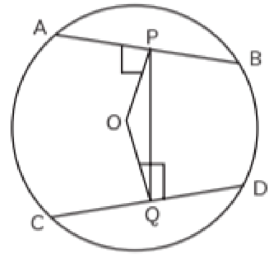
$\Rightarrow a + \angle QPS = 180^\circ$

$\Rightarrow a + 50^\circ = 180^\circ$

$\Rightarrow a = 180^\circ - 50^\circ$

$\Rightarrow a = 130^\circ$

70. **In fig., AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD respectively. If $\angle POQ = 150^\circ$, find $\angle APQ$.**



[Delhi Gov. QB 2022]

Ans. Since AB = CD (equal chords) so, their distance from centre must be equal.

So, OP = OQ

Now, in ΔPOQ,

(1) $\angle OPQ = \angle OQP$

(2) $\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$\Rightarrow 2\angle OPQ + 150^\circ = 180^\circ$

$\Rightarrow \angle OPQ = 15^\circ$

Also, since P is the mid-point of AB

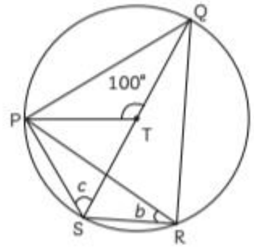
$OP \perp AB \Rightarrow \angle APO = 90^\circ$

Now, $\angle APQ = \angle APO - \angle OPQ = 90^\circ - 15^\circ = 75^\circ$

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

71. In the quadrilateral, PQRS inscribed in a circle with centre T, find the value of *b* and *c*.



Ans. On the line SQ,

$$\angle PTQ + \angle PTS = 180^\circ \quad \text{[Linear pair]}$$
$$100^\circ + \angle PTS = 180^\circ$$
$$\angle PTS = 180^\circ - 100^\circ$$
$$\angle PTS = 80^\circ \quad ...(i)$$

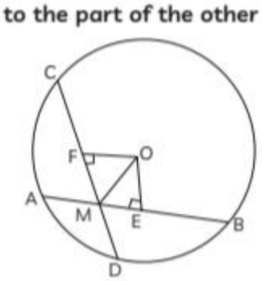
Now, from segment PS, the angle made on the centre is double the angle made by the same segment on the circumference.

$$\angle PTS = 2\angle PRS$$
$$80^\circ = 2 \times b$$
$$b = 40^\circ$$

Similarly,

$$\angle PTQ = 2\angle PSQ \text{ [PQ is common]}$$
$$100^\circ = 2 \times c$$
$$c = \frac{100^\circ}{2}$$
$$c = 50^\circ$$

72. If two equal chords of a circle intersect, prove that parts of one chord are separately equal to the part of the other chord.



[NCERT Exemplar]

Ans. AB and CD are two equal chords of a circle with centre O, intersect each other at M.

To Prove:

(i) MB = MC

(ii) AM = MD

AB is a chord and OE ⊥ AB from the centre O. The perpendicular drawn from the centre to a chord bisects the chord.

$$AE = \frac{1}{2} AB$$

Similarly,

$$FD = \frac{1}{2} CD$$

$$AB = CD \quad \text{[Given]}$$

So, $\frac{AB}{2} = \frac{CD}{2}$

$$AE = FD \quad ....(i)$$

Equal chords are equidistant from the centre

$$OE = OF \quad [AB = CD]$$

In, ΔMOE and ΔMOF

$$OM = OM \quad \text{[Common]}$$
$$OE = OF$$
$$\angle E = \angle F \quad [90^\circ \text{ each}]$$

ΔMOF ≅ ΔMOE [By RHS Congruency Rule]

Therefore, ME = MF [By CPCT] ......(ii)

Subtract eqs. (ii) from (i), we get

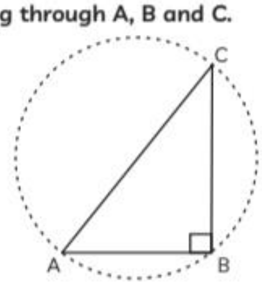
$$AE - ME = FD - MF$$
$$AM = MD$$

Again,

$$AB = CD$$
$$AM = MD$$
$$AB - AM = CD - MD$$

So, MB = MC

73. If in a ΔABC, AB = 12 cm, BC = 16 cm and AB ⊥ BC, then find the radius of the circle passing through A, B and C.



[Delhi Gov. QB 2022]

Ans. Given: A, B and C are points such that AB = 12 cm and BC = 16 cm and BC ⊥ AB.

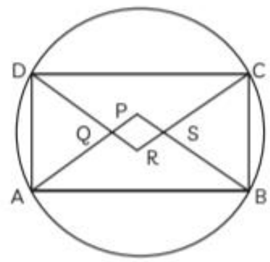
BC ⊥ AB *i.e.*, ∠ABC = 90° and ΔABC is a right one with AC as hypotenuse.

$\therefore$ AC = $\sqrt{AB^2+BC^2} = \sqrt{12^2+16^2}$ cm = 20 cm.

So, the circle passing through A, B and C will have its diameter as AC.

$\therefore$ Its radius = $\frac{1}{2} \times 20$ cm = 10 cm.

74. A quadrilateral is inscribed in a circle and the angle bisector of the cyclic quadrilateral makes a quadrilateral PQRS as shown in the figure below. If $\angle P = 70^\circ$, find $\angle R$.



Ans. Sum of the angles of a triangle is 180°.

In ΔAPB and ΔCRD

$$\angle APB + \angle PAB + \angle PBA = 180^\circ$$
$$\angle CRD + \angle RCD + \angle RDC = 180^\circ$$

Now,

$$\angle APB + \frac{1}{2}\angle A + \frac{1}{2}\angle B = 180^\circ \quad \text{..... (i)}$$

[AP and BP are bisectors of $\angle A$ and $\angle B$ respectively]

Also,

$$\angle CRD + \frac{1}{2}\angle C + \frac{1}{2}\angle D = 180^\circ \quad \text{..... (ii)}$$

[CR and DR are bisectors of $\angle C$ and $\angle D$ respectively]

Add eqs. (i) and (ii)

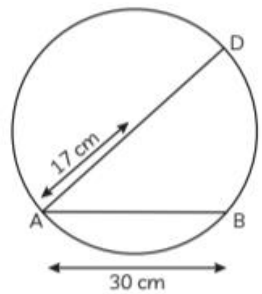
$$\angle APB + \frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle CRD + \frac{1}{2}\angle C + \frac{1}{2}\angle D = 360^\circ$$
$$\angle APB + \angle CRD + \frac{1}{2}[\angle A + \angle B + \angle C + \angle D] = 360^\circ$$
$$\angle APB + \angle CRD + \frac{1}{2}[(\angle A + \angle C) + (\angle B + \angle D)] = 360^\circ$$
$$\angle APB + \angle CRD + \frac{1}{2}[180^\circ + 180^\circ] = 360^\circ$$
$$\angle APB + \angle CRD + 180^\circ = 360^\circ$$
$$\angle APB + \angle CRD = 180^\circ$$

Hence, PQRS is a cyclic quadrilateral

So, $70^\circ + \angle CRD = 180^\circ$

$$\angle CRD = 180^\circ - 70^\circ$$
$$\angle CRD = 110^\circ$$

75. AD is a diameter of a circle and AB is a chord if AD = 34 cm, AB = 30 cm, then find the distance of AB from the centre of circle.

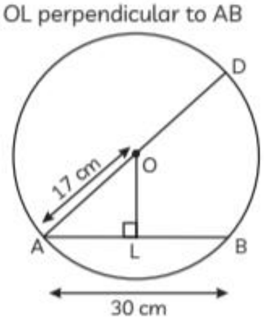


[Delhi Gov. QB 2022]

Ans. It is given that

$$AD = 34 \text{ cm}$$
$$AB = 30 \text{ cm}$$

Construct OL perpendicular to AB



$$AL = LB = \frac{1}{2} AB = \frac{1}{2}(30) = 15 \text{ cm}$$

In triangle OLA,

Using Pythagoras theorem

$$OA^2 = OL^2 + AL^2$$

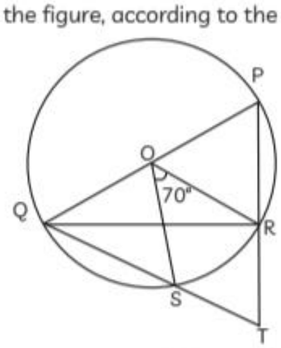
Substituting the values

$$17^2 = OL^2 + 15^2$$
$$OL^2 = 289 - 225 = 64$$
$$OL = 8 \text{ cm}$$

Therefore, the distance of AB from the centre of the circle is 8 cm.

76. PQ is the diameter of the circle and PR and QS are the chords. Extended QS and PR meet at T outside the circle. Join QR. If $\angle SOR = 70^\circ$, find complementary $\angle STR$.

Ans. Draw the figure, according to the question.



Here, $\angle SOR = 70^\circ$

Now, $\angle RQS = \frac{1}{2} \angle SOR$

[Arc SR is common]

$\angle RQS = \frac{1}{2} \times 70^\circ$

$\angle RQS = 35^\circ$

Also, $\angle QRP = 90^\circ$

[Angle in a semicircle]

Now, in ΔTQR,

$\angle QRT + \angle RQT + \angle QTR = 180^\circ$

[Angle sum property]

$90^\circ + 35^\circ + \angle QTR = 180^\circ$

$\angle QTR = 180^\circ - 125^\circ$

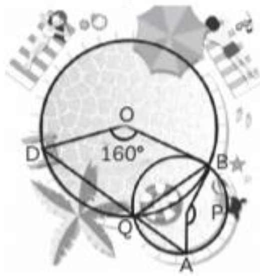
$\angle QTR = 55^\circ$

Hence, $\angle STR = \angle QTR = 55^\circ$

So, complimentary angle of $\angle STR$

$= 90^\circ - 55^\circ = 35^\circ$

77. During a trip with parents three siblings planned to play in a circular swimming club together, as shown in figure. Point O is the centre of bigger swimming pond and point P is the centre of smaller swimming pool. Find the measure of $\angle APB$, if $\angle BOD = 160^\circ$.



Ans. Reflex angle $\angle BOD = (360^\circ - 160^\circ)$

$= 200^\circ$

Now, we know that the angle made by arc on the centre is double to that of the circumference.

Therefore, reflex $\angle BOD = 2\angle BQD$

$\angle BQD = \frac{1}{2} \times \angle BOD$

$= \frac{1}{2} \times 200^\circ = 100^\circ$

Now, DA is a straight line

So, $\angle AQB = 180^\circ - \angle BQD$

[Linear pair]

$\angle AQB = 180^\circ - 100^\circ$

$\angle AQB = 80^\circ$

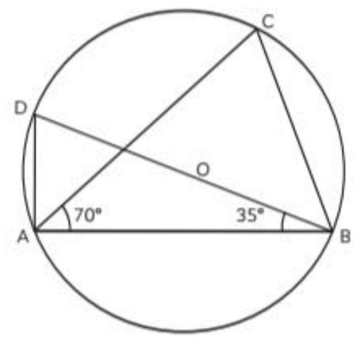
Hence, $\angle APB = 2\angle AQB$

[Arc AB is common]

$= 2 \times 80^\circ$

$= 160^\circ$

78. In the given figure, O is the centre of the circle. If $\angle ABD = 35^\circ$ and $\angle BAC = 70^\circ$, find $\angle ACB$.



[Delhi Gov. QB 2022]

Ans. We know that BD is the diameter of the circle.

Angle in a semicircle is a right angle

$\therefore$ $\angle BAD = 90^\circ$

Consider ΔBAD

Using the angle sum property

$\angle ADB + \angle BAD + \angle ABD = 180^\circ$

By substituting the values

$\angle ADB + 90^\circ + 35^\circ = 180^\circ$

$\angle ADB = 180^\circ - 90^\circ - 35^\circ$

$\angle ADB = 180^\circ - 125^\circ$

So, we get

$\angle ADB = 55^\circ$

We know that the angle in the same segment of a circle are equal.

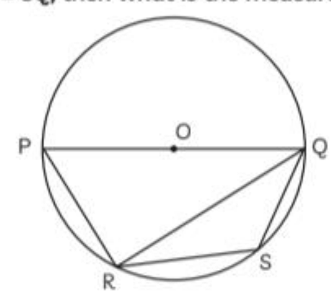
$\angle ACB = \angle ADB = 55^\circ$

So, we get

$\angle ACB = 55^\circ$

Therefore, $\angle ACB = 55^\circ$

79. In a circle with centre O, if PQ is diameter and RS is a chord such that $\angle PQR = 36^\circ$ and RS = SQ, then what is the measure of $\angle RQS$?



Ans. Here, PQ is diameter and the angle in a semicircle is 90°.

$\angle PRQ = 90^\circ$

$\angle PQR = 36^\circ$ [Given]

In ΔPQR,

$\angle PRQ + \angle PQR + \angle RPQ = 180^\circ$

[Angle sum property]

$90^\circ + 36^\circ + \angle RPQ = 180^\circ$

$\angle RPQ = 180^\circ - 90^\circ - 36^\circ$

$\angle RPQ = 54^\circ$ or, $\angle P = 54^\circ$

In cyclic quadrilateral PRSQ,

$\angle P + \angle S = 180^\circ$

[The opposite angles of a cyclic quadrilateral is supplementary]

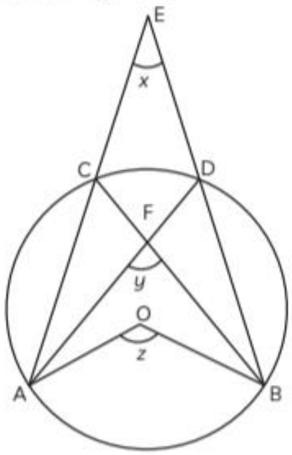
$$54^\circ + \angle S = 180^\circ$$
$$\angle S = 180^\circ - 54^\circ$$
$$\angle S = 126^\circ$$

Since, $RS = SQ$ [Given]

Therefore, $\angle RQS = \angle QRS$ ..... (i)

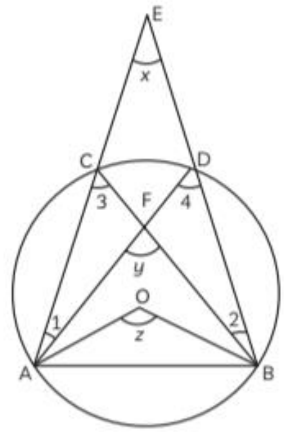
$$\angle RQS + \angle QRS + \angle RSQ = 180^\circ$$
$$\angle RQS + \angle RQS + 126^\circ = 180^\circ \quad \text{[From eq. (i)]}$$
$$\angle RQS = \frac{180^\circ - 126^\circ}{2} = \frac{54^\circ}{2}$$
$$\angle RQS = 27^\circ$$

80. In the given figure, O is the centre of a circle prove that $\angle x + \angle y = \angle z$.



[Delhi Gov. QB 2022]

Ans. In ΔACF, side $\angle$CF is produced to B.



$\therefore \quad \angle y = \angle 1 + \angle 3$ ...(i)

[Ext. angle = sum of int. opp. angle]

In ΔAED, side ED is produced to B.

$\therefore \quad \angle 1 + \angle x = \angle 4$ ...(ii)

Add eqs. (i) and (ii), we have

$$\angle 1 + \angle x + \angle y = \angle 1 + \angle 3 + \angle 4$$
$$\Rightarrow \quad \angle x + \angle y = \angle 3 + \angle 4$$
$$= 2\angle 3$$

[$\because$ $\angle 4 = \angle 3$, angles in the same segment]

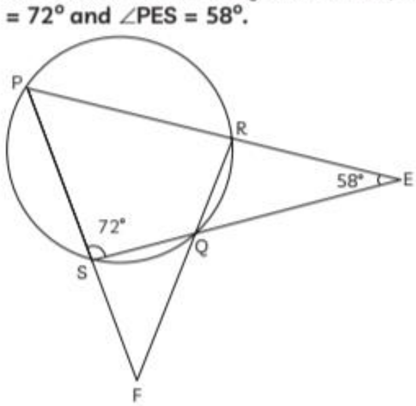
$$= \angle z \quad [\because \angle AOB = 2\angle ACB]$$
$$\angle x + \angle y = \angle z$$

Hence, proved.

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

81. In the figure shown below, side PR and SQ of a cyclic quadrilateral PRQS arc produced to meet at E and sides PS and RQ are produced to meet at F. Find $\angle$RQS and $\angle$PFR, if $\angle$PSQ = 72° and $\angle$PES = 58°.



Ans. Here, in ΔPES,

$$\angle EPS + \angle PES + \angle ESP = 180^\circ$$

[Angle sum property]

$$\angle EPS + 58^\circ + 72^\circ = 180^\circ$$
$$\angle EPS = 180^\circ - 130^\circ$$
$$\angle EPS = 50^\circ$$

Or, $\angle RPS = 50^\circ$

Now, $\angle PSQ + \angle PRQ = 180^\circ$

[Opposite angles of a cyclic quadrilateral are supplementary]

$$72^\circ + \angle PRQ = 180^\circ$$
$$\angle PRQ = 180^\circ - 72^\circ$$
$$\angle PRQ = 108^\circ$$

Similarly,

$$\angle RPS + \angle RQS = 180^\circ$$

$$50^\circ + \angle RQS = 180^\circ$$
$$\angle RQS = 180^\circ - 50^\circ$$
$$\angle RQS = 130^\circ$$

Now, in ΔPRF,

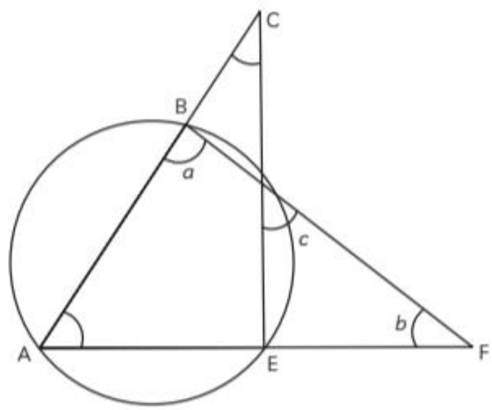
$$\angle RPF + \angle PRF + \angle PFR = 180^\circ$$

[Angle sum property]

$$108^\circ + 50^\circ + \angle PFR = 180^\circ$$
$$158^\circ + \angle PFR = 180^\circ$$
$$\angle PFR = 180^\circ - 158^\circ$$
$$\angle PFR = 22^\circ$$

Hence, $\angle PFR = 22^\circ$ and $\angle RQS = 130^\circ$

82. In the given figure, determine *a*, *b* and *c*, if $\angle BCD = 43^\circ$ and $\angle BAF = 62^\circ$.



[Delhi Gov. QB 2022]

Ans. Now, $\angle ACE = 43^\circ$ and $\angle CAF = 62^\circ$ [Given]

In ΔAEC,

$\therefore \quad \angle ACE + \angle CAE + \angle AEC = 180^\circ$

$\Rightarrow \quad 43^\circ + 62^\circ + \angle AEC = 180^\circ$

$\Rightarrow \quad 105^\circ + \angle AEC = 180^\circ$

$\Rightarrow \quad \angle AEC = 180^\circ - 105^\circ = 75^\circ$

Now, $\quad \angle ABD + \angle AED = 180^\circ$

[Opposite angles of a cyclic quadrilateral and $\angle AED = \angle AEC$]

$\Rightarrow \quad a + 75^\circ = 180^\circ$

$\Rightarrow \quad a = 180^\circ - 75^\circ$

$\Rightarrow \quad a = 105^\circ$

$\angle EDF = \angle BAF$

$\therefore \quad c = 62^\circ$

[Angles in the alternate segments]

In ΔBAF,

$a + 62^\circ + b = 180^\circ$

$\Rightarrow \quad 105^\circ + 62^\circ + b = 180^\circ$

$\Rightarrow \quad 167^\circ + b = 180^\circ$

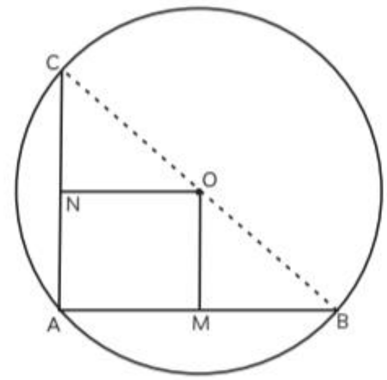
$\Rightarrow \quad b = 180^\circ - 167^\circ = 13^\circ$

Hence, $a = 105^\circ$, $b = 13^\circ$ and $c = 62^\circ$

83. AB and AC are two chords of a circle of radius *r* such that AB = 2AC. If *p* and *q* are the distance of AB and AC from the centre. Prove that $4q^2 = p^2 + 3r^2$.

[Delhi Gov. QB 2022]

Ans. Consider O as the centre of the circle with radius *r*.



So, we get

$$OB = OC = r$$

Consider $AC = x$ and $AB = 2x$

We know that $OM \perp AB$

So, we get

$$OM = p$$

We know that $ON \perp AC$

So, we get

$$ON = q$$

Consider ΔOMB

Using the Pythagoras theorem

$$OB^2 = OM^2 + BM^2$$

We know that the perpendicular from the centre of the circle bisects the chord.

So, we get

$$r^2 = p^2 + [(\frac{1}{2})\ AB]^2$$

It can be written as

$$r^2 = p^2 + \frac{1}{4} \times 4x^2$$

So, we get

$$r^2 = p^2 + x^2 \qquad \text{..... (i)}$$

Consider ΔONC

Using the Pythagoras theorem

$$OC^2 = ON^2 + CN^2$$

We know that the perpendicular from the centre of the circle bisects the chord.

So, we get

$$r^2 = q^2 + [(\frac{1}{2})\ AC]^2$$

It can be written as

$$r^2 = q^2 + \frac{1}{4} \times x^2$$

$$r^2 = q^2 + \frac{x^2}{4}$$

We get

$$q^2 = r^2 - \frac{x^2}{4}$$

Multiplying the equation by 4

$$4q^2 = 4r^2 - x^2$$

Substituting equation (i)

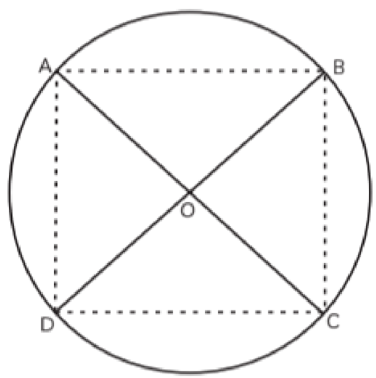
$$4q^2 = 4r^2 - (r^2 - p^2)$$

So, we get

$$4q^2 = 3r^2 + p^2$$

Therefore, it is proved that $4q^2 = p^2 + 3r^2$.

84. AC and BD are chords of a circle which bisect each other. Prove that AC and BD are diameters of the circle and ABCD is a rectangle. [Diksha]



Ans. Let two chords AB and CD are intersecting each other at point O.

In ΔAOB and ΔCOD,

OA = OC [Given]

OB = OD [Given]

∠AOB = ∠COD [By Vertically opposite angles]

ΔAOB ≅ ΔCOD [SAS congruence rule]

AB = CD [By CPCT]

Similarly, it can be proved that ΔAOD ≅ ΔCOB

∴ AD = CB [By CPCT]

Since in quadrilateral ABCD, opposite sides are equal in length, ABCD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

However, ∠A + ∠C = 180° [ABCD is a cyclic quadrilateral]

⇒ ∠A + ∠A = 180°

⇒ 2∠A = 180°

⇒ ∠A = 90°

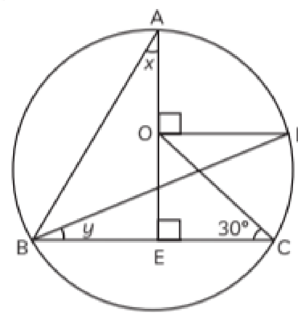
As ABCD is a parallelogram and one of its interior angles is 90°.

Therefore, it is a rectangle.

∠A is the angle subtended by chord BD. And as ∠A = 90°, therefore, BD should be the diameter of the circle.

Similarly, AC is the diameter of the circle.

85. In figure, O is the centre of the circle, ∠BCO = 30°, AE ⊥ BC and DO ⊥ AE. Find x and y.



[Delhi Gov. QB 2022]

Ans. In the given figure, OD is parallel to BC.

∴ ∠BCO = ∠COD [Alternate interior angles]

⇒ ∠COD = 30° ...... (i)

We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by it on the remaining part of the circle.

Here, arc CD subtends ∠COD at the centre and ∠CBD at B on the circle.

∴ ∠COD = 2∠CBD

⇒ $\angle CBD = \frac{30°}{2} = 15°$

∴ $y = 15°$ ...... (ii) [From (i)]

Also, arc AD subtends ∠AOD at the centre and ∠ABD at B on the circle.

∴ ∠AOD = 2∠ABD

⇒ $\angle ABD = \frac{90°}{2} = 45°$ ...... (iii)

In ΔABE,

$x + y$ + ∠ABD + ∠AEB = 180°

⇒ $x + 15° + 45° + 90° = 180°$

⇒ $x = 180° - (90° + 15° + 45°)$

⇒ $x = 180° - 150°$

⇒ $x = 30°$ [Sum of the angles of a triangle]

[From eqs. (ii) and (iii)]

Hence, $x = 30°$ and $y = 15°$.